A General Pseudo-Conservation Law for a Polling System with Time-Limited Service Discipline

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Abstract We consider a cyclic-service queueing system (polling system) with time-limited service, in which the length of a service period for each queue is controlled by a timer, i.e., the server serves customers until the timer expires or the queue becomes empty, whichever occurs first, and then proceeds to the next queue. The customer whose service is interrupted due to the timer expiration is attended according to the non-preemptive service discipline. For the cyclic-service system with structured batch Poisson arrivals ($M^X/G/1$) and an exponential timer, we derive a psuedo-conservation law and an exact mean waiting time formula for the symmetric system. Furthermore we provide a general psuedo-conservation law for exponential time-limited service and some basic services.

Keywords: Polling system, structured batch Poisson arrival, exponential time-limited service, non-preemptive discipline, psuedo-conservation law.

1 Introduction

In cyclic-service queueing systems (polling systems) with non-zero switch-over times, Watson (1984) and Ferguson and Aminetzah (1985) have first found the well-known pseudo-conservation law given by a weighted sum of mean waiting times. Using the stochastic decomposition of the workload in vacation systems, pseudo-conservation laws have systematically been derived by Boxma and Groenendijk (1987) for basic service disciplines, such as exhaustive, gated, one-limited (non-exhaustive) and one-decrementing (semi-exhaustive) services. After them, these pseudo-conservation laws have been extended to various service disciplines and a compound Poisson process with correlated arrivals as reviewed in survey articles by Takagi (1997). These pseudo-conservation laws can be used to obtain simple and yet accurate approximations for the individual mean waiting times in asymmetric systems and also useful for optimization problems in flexible service disciplines with controllable parameters, e.g., Boxma, Levy and Weststrate (1990) and Katayama, Kobayashi and Nakagawa (2003).

On the other hand, time-limited service polling systems have gained much attention in view of both the applications and the theoretical analysis. The term time-limited service refers to the fact that the server serves a queue only up to an amount of time controlled by a timer during each service period, that is, the server serves waiting customers (messages or packets) until the timer expires or the queueing buffer becomes empty, whichever occurs first, and then proceeds to the next queue. The limited time is also called the maximum server attendance (MSA) time by Leung and Eisenberg (1990). The time-limited service disciplines are classified as exhaustive (or non-gated) and gated time-limited services and furthermore, as non-preemptive and preemptive-resume service disciplines with respect to the interrupted service caused by timer expiration as in Katayama (2001). The main merit of the time-limited service is that the MSA time can be arbitrarily adjusted. Such a flexible schedule is effective for the performance optimization, and has a potential applicability to communication systems with multiple grades of service requirements in multi-media broad-band networks.

In this paper, we will derive a pseudo-conservation law for an $M^X/G/1$-type cyclic-service system with an exponential time-limited service, which is an extended result of Katayama, Kobayashi and Miura (2001) and will provide a general pseudo-conservation law for mixed basic service disciplines, which is also an extension of the recent result of Katayama and Kobayashi (2002). The rest of the paper is organized as follows: In Section 2, we describe the model and notation. After pleliminaries in Section 3, we derive a pseudo-conservation law, and give an explicit mean waiting time formula in Section 4. We provide a general pseudo-conservation law for the cyclic-service system with time-limited service and some basic service disciplines in Section 5.

2 Model and Notation

We consider an $M^X/G/1$ cyclic-service queueing system with N infinite capacity buffers which are denoted by $Q_1, Q_2, \cdots, Q_N$ and assume that the system has a Poisson arrivals process at rate λ such that each arrival contains G_i customers in $Q_i, i = 1, 2, \cdots, N$, simultaneously. The generating functions (GFs) of the joint probability distribution, $g(k_1, k_2, \cdots, k_N) := \Pr\{G_1 = k_1, G_2 = k_2, \cdots, G_N = k_N\}$, $k_i \geq 0$, $i = 1, 2, \cdots, N$ and the marginal distribution are denoted by, respectively,

$$G(z_1, z_2, \cdots, z_N) := \boldsymbol{E}[z_1^{G_1}, z_2^{G_2}, \cdots, z_N^{G_N}] = G(z),$$
$$G_i(x) := G(z_1 = 1, \cdots, z_i = x, \cdots, z_N = 1).$$

Some moments of the joint distribution for $\{G_i\}$ are denoted by

$$g_i := \left[\frac{\partial G(z)}{\partial z_i}\right]_{z=\mathbf{1}} = \boldsymbol{E}[G_i], \quad g_{i,j} := \left[\frac{\partial^2 G(z)}{\partial z_i \partial z_j}\right]_{z=\mathbf{1}} = \boldsymbol{E}[G_i G_j] \qquad \text{for } i \neq j,$$
$$g_i^{(2)} := \left[\frac{\partial^2 G(z)}{\partial z_i^2}\right]_{z=\mathbf{1}} = \boldsymbol{E}[G_i(G_i - 1)],$$

where the $z = \mathbf{1}$ stands for $(z_1 = 1, \cdots, z_i = 1, \cdots, z_N = 1)$. The maximum length of a service period of a single server at $Q_i, i = 1, 2, \cdots, N$ is limited by a given time T_i called the MSA time, in other words, the server serves the customers in Q_i until either the time limit expires, or the queue becomes empty, whichever occurs first, and then proceeds to Q_{i+1} mod (N), where customers arriving at currently in service can possibly be served in the same service period, i.e., exhaustive service discipline. Furthermore, the service on the customer being served is completed during the current service period, i.e., non-preemptive discipline. We assume that the MSA time T_i for $Q_i, i = 1, 2, \cdots, N$ is exponentially distributed with mean $\overline{T}_i := 1/\alpha_i$. The Laplace-Stieltjes transform (LST) and the distribution function (DF) of the MSA time $T_i, i = 1, 2, \cdots, N$ are denoted by $T_i^*(s) := \alpha_i/(s + \alpha_i)$ and $T_i(t)$, respectively. The time-limited schedule can be parametrized by a vector of $(\overline{T}_1, \overline{T}_2, \cdots, \overline{T}_N)$. The LST of the DF, the mean and the second moment of the service time $H_i, i = 1, 2, \cdots, N$ of a customer at Q_i are denoted by $H_i^*(s)$, h_i and $h_i^{(2)}$, respectively. Each arrival is also considered as a supercustomer whose service time (B) has the LST $B^*(s)$ of the DF, the mean and the second moment given by, respectively,

$$B^*(s) := G(H_1^*(s), H_2^*(s), \cdots, H_N^*(s)),$$

$$b := \sum_{i=1}^{N} g_i h_i,$$
$$b^{(2)} := \sum_{i=1}^{N} \left(g_i h_i^{(2)} + g_i^{(2)} h_i^2 \right) + 2\sum_{i=2}^{N} h_i \sum_{j=1}^{i-1} g_{i,j} h_j.$$

The total load offered to the system is then given by

$$\hat{\rho} := \lambda b = \sum_{i=1}^{N} \rho_i,$$
$$\rho_i := \lambda_i h_i, \quad \lambda_i := \lambda g_i, \quad i = 1, 2, \cdots, N.$$

The LST of the DF, the mean and the second moment of the switch-over time D_i, $i = 1, 2, \cdots, N$ needed by the server to switch from Q_i to Q_{i+1} are denoted by $D_i^*(s)$, d_i and $d_i^{(2)}$, respectively. The switch-over times are independent of the arrival and service processes. The mean and the variance of the total switch-over time during a cycle of the server are then given by, respectively,

$$\overline{D} := \sum_{i=1}^{N} d_i, \quad \sigma_D^2 := \sum_{i=1}^{N} \left(d_i^{(2)} - d_i^2 \right).$$

We refer to an instant the server arrives at Q_i from Q_{i-1} as a *polling instant* of Q_i. Furthermore, we define the *polling cycle time* (C) as the time between the server's visit to the same queue in successive cycles, the *service period* (S_i) of Q_i as the time between the arrival of the server at Q_i and his subsequent departure from Q_i, and the *intervisit time* (I_i) for Q_i as the time between the server's departure from Q_i and the next polling instant of Q_i.

Remark 2.1. We do not consider a batch that contains no customers at all, i.e. $g(0, 0, \cdots, 0) = 0$. Some special cases are as follows; If each arrival contains only customers for a single queue, we have $g_{i,j} = 0$ for $i \neq j$, $N \geq i, j \geq 1$. Furthermore, if each arrival contains a single customer, we have $g_i = 1$ and $g_i^{(2)} = 0$ for $N \geq i \geq 1$. If the number of customers contained in each arrival are independent for different queues, we have $g_{i,j} = g_i g_j$ for $i \neq j$, $N \geq i, j \geq 1$, see Sidi and Segall (1983).

3 Preliminaries

The non-preemptive, time-limited schedule is closely related to the Bernoulli schedule with parameters (p_1, p_2, $\cdots$, p_N). From correspondence of the time-limited service to the Bernoulli schedule, we have the following results:

$$p_i := \Pr\{T_i > H_i\} = H_i^*(\alpha_i), \quad \overline{p}_i := 1 - H_i^*(\alpha_i). \tag{1}$$

We define the following LSTs and GFs for $i = 1, 2, \cdots, N$:

$$F_i^*(s) := \boldsymbol{E}[e^{-sH_i} | T_i > H_i] = \frac{H_i^*(s + \alpha_i)}{H_i^*(\alpha_i)},$$
$$\overline{F}_i^*(s) := \boldsymbol{E}[e^{-sH_i} | T_i \leq H_i] = \frac{H_i^*(s) - H_i^*(s + \alpha_i)}{1 - H_i^*(\alpha_i)}, \tag{2}$$
$$Q_{F_i}(x) := F_i^*(\lambda - \lambda G_i(x)), \quad Q_{\overline{F}_i}(x) := \overline{F}_i^*(\lambda - \lambda G_i(x)),$$
$$Q_{f_i}(x) := p_i F_i^*(\lambda - \lambda G_i(x)), \quad Q_{\overline{f}_i}(x) := \overline{p}_i \overline{F}_i^*(\lambda - \lambda G_i(x)),$$
$$Q_{H_i}(x) := H_i^*(\lambda - \lambda G_i(x)).$$

Further, we define the following random variables for $i = 1, 2, \cdots, N$:

K_i := the number of customers at the polling instant at Q_i,

N_i := the number of customers served during S_i,
L_i := the number of remaining customers in Q_i when the server leaves Q_i.
Then, we have

$$\boldsymbol{E}(L_i) = \boldsymbol{E}(K_i) - (1-\rho_i)\boldsymbol{E}(N_i), \tag{3}$$

which can be derived from the following known equations:

$$\begin{aligned} &\boldsymbol{E}(K_i) + \lambda_i \boldsymbol{E}(S_i) = \boldsymbol{E}(N_i) + \boldsymbol{E}(L_i), \quad \boldsymbol{E}(S_i) = h_i \boldsymbol{E}(N_i), \\ &\boldsymbol{E}(S_i) = \rho_i \boldsymbol{E}(C), \quad \boldsymbol{E}(I_i) = (1-\rho_i)\boldsymbol{E}(C), \quad \boldsymbol{E}(C) = \overline{D}/(1-\rho). \end{aligned} \tag{4}$$

In the next section, we will use Lemma 1 on the following GFs defined by: For $m \geq 0$,

$P_i(x)$:= GF of the DF $\{p_i(m)\}$ of the numbers (m) of customers in Q_i at an arbitrary epoch,
$P_i^-(x)$:= GF of the DF $\{p_i^-(m)\}$ of the numbers of customers in Q_i just before an arrival epoch,
$\Pi_i(x)$:= GF of the DF $\{\pi_i(m)\}$ of the numbers of customers in Q_i just after a departure epoch of a customer from the system.
$\Pi_i^-(x)$:= GF of the DF $\{\pi_i^-(m)\}$ of the numbers of customers in Q_i just before a departure epoch of a customer from the system.

Lemma 1.

$$\Pi_i(x) = P_i(x)R_i(x), \qquad R_i(x) := \frac{1-G_i(x)}{g_i(1-x)}. \tag{5}$$

Proof: Let a set $S_m := \{m+1, m+2, \cdots\}$ and $S_m^C := \{0, 1, \cdots, m\}$ for $m \geq 0$ with respect to the numbers of customers in Q_i. Then using the exit rate (r_{out}) from the set S_m and the entry rate (r_{in}) into S_m, that is, from the discrete-state level-crossing analysis regarding the state $\{m+1\}$, we get

$$\begin{aligned} r_{out} &:= \lambda_{down}\pi_i^-(m+1) = \lambda g_i \pi_i(m), \\ r_{in} &:= \lambda_{up}\sum_{j=0}^{m} p_i^-(j) \sum_{k=m-j+1}^{\infty} \Pr\{G_i = k\} = \lambda \sum_{j=0}^{m} p_i(j)\left[1 - \sum_{k=0}^{m-j}\Pr\{G_i = k\}\right], \end{aligned}$$

where $\lambda_{down}(\lambda_{up})$ is the long run rate of downward (upward) jumps, e.g. see Shanthikumar (1982), and we have used the PASTA (Poisson arrivals see time averages) property for the second equation with λ_{up}. Equating the exit and entry rates, we obtain

$$g_i \pi_i(m) = \sum_{j=0}^{m} p_i(j)\left[1 - \sum_{k=0}^{m-j}\Pr\{G_i = k\}\right]. \tag{6}$$

Finally, taking the generating function of (6) we get (5). □

The GF $R_i(x)$ represents the GF for the backward recurrence time in the discrete-time renewal process, where the interval between two successive renewal points is given by G_i. A probabilistic interpretation for (5) is given as follows: First of all, we assume the FIFO discipline for Q_i. Then, $P_i(x)R_i(x)$ represents the GF for the number of all customers placed before an arbitrary tagged customer chosen randomly from an arriving batch in Q_i when the tagged customer has arrived at Q_i because of the PASTA property, while $\Pi_i(x)$ the GF for the number of customers being behind the tagged customer in Q_i just after the tagged customer's departure epoch. The both GFs should be equal in steady state. Here, note that (5) holds for any service mechanism with non-batch and non-preemptive services, since the number of customers in Q_i is independent of service disciplines as FIFO, LIFO, etc. Eq. (5) for the time-limited service polling system is a generalized result for the $M^X/G/1$ single-queue (Chaudhry (1979)), and also follows from the result derived by Takine and Takahashi (1998) as a special case of a batch Markovian arrival process (BMAP).

4 Queueing Analysis

We will derive Theorem 1 for the time-limited service system using the GFs formulated on both service-beginning and customer-departure epochs. In what follows, an epoch is a polling instant, a service completion or a service beginning for a customer in Q_i. We consider a sequence of pairs of random variables $(L_n, J_n), n = 1, 2, \cdots$ defined as follows: L_n denotes the number of customers at the n-th epoch, while $J_n = 0$ if the epoch marks a polling instant of Q_i, $J_n = 1$ if the epoch marks a service completion of a customer in Q_i and $J_n = 2$ if the epoch marks a service beginning for each customer.

Theorem 1 (Pseudo-conservation law).
For a stable $M^X/G/1$ cyclic-service system with an exponential time-limited service specified by a vector $(\overline{T}_1, \overline{T}_2, \cdots, \overline{T}_N)$, the following relationship among the mean waiting times holds:

$$\sum_{i=1}^{N}\rho_i\left[1-\frac{\lambda_i\overline{D}}{1-\hat{\rho}}(1-H_i^*(\alpha_i))\right]\boldsymbol{E}(W_i)=\frac{\lambda}{2(1-\hat{\rho})}\sum_{i=1}^{N}(\rho g_i h_i^{(2)}+g_i^{(2)}h_i^2)$$
$$+\frac{\lambda}{1-\hat{\rho}}\sum_{i=2}^{N}h_i\sum_{j=1}^{i-1}g_{i,j}h_j+\frac{\hat{\rho}\sigma_D^2}{2\overline{D}}+\frac{\overline{D}}{2(1-\hat{\rho})}\left[\hat{\rho}-\sum_{i=1}^{N}\rho_i^2\right] \tag{7}$$
$$+\frac{\lambda\overline{D}}{2(1-\hat{\rho})}\sum_{i=1}^{N}h_i\left[2\rho_i g_i+2\lambda_i g_i\frac{d}{d\alpha_i}H_i^*(\alpha_i)+g_i^{(2)}(1-H_i^*(\alpha_i))\right].$$

Proof: First of all, we define the following GFs:

$$\Phi_i(x) := \lim_{n\to\infty}\boldsymbol{E}[x^{L_n}|J_n=0],$$
$$\Pi_i(x) = \lim_{n\to\infty}\boldsymbol{E}[x^{L_n}|J_n=1],$$
$$\Pi_i^+(x) := \lim_{n\to\infty}\boldsymbol{E}[x^{L_n}|J_n=2],\qquad (=\sum_{k=1}^{\infty}\pi_i^+(k)x^k),$$

where $\Pi_i^+(0)\equiv 0$. Then we obtain a functional relationship between $\Pi_i(x)$ and $\Pi_i^+(x)$,

$$\Pi_i(x)=\Pi_i^+(x)\left[Q_{f_i}(x)+Q_{\overline{f}_i}(x)\right]\frac{1}{x}=\Pi_i^+(x)Q_{H_i}(x)\frac{1}{x} \tag{8}$$

and a functional relationship with $\Pi_i^+(x)$,

$$\Pi_i^+(x)=\kappa\left[\Phi_i(x)-\Phi_i(0)\right]+\Pi_i^+(x)Q_{f_i}(x)\frac{1}{x}-\pi_i^+(1)xQ_{f_i}(0)\frac{1}{x}, \tag{9}$$

where

$$\kappa := \lim_{n\to\infty}\Pr\{J_n=0\}/\Pr\{J_n=2\}=(1-\hat{\rho})/\lambda_i\overline{D}. \tag{10}$$

Note here that $1/\kappa$ is nothing but the mean number of customers served in one service-period at Q_i, i.e. $1/\kappa = E(N_i)$. Combining (8), (9) and Lemma 1, we obtain finally

$$(x - Q_{f_i}(x))R_i(x)P_i(x) = \frac{(1-\hat{\rho})Q_{H_i}(x)}{\lambda_i \overline{D}} \left[\Phi_i(x) + \frac{\lambda_i \overline{p}_i \overline{D}}{1-\hat{\rho}} - 1 \right]. \tag{11}$$

Therefore, taking the first derivative of both sides of (11) with respect to x and applying Little's formula to $P_i'(1)$ lead to

$$\Phi_i'(1) = \frac{\lambda_i \overline{D}}{1-\hat{\rho}} \left[1 + \lambda_i \frac{d}{d\alpha_i} H_i^*(\alpha_i) + (1 - H_i^*(\alpha_i)) \left\{ \frac{g_i^{(2)}}{2g_i} + \lambda_i \boldsymbol{E}(W_i) \right\} \right]. \tag{12}$$

Furthermore, it follows from the decomposition theorem for vacation systems that

$$\sum_{i=1}^{N} \rho_i \boldsymbol{E}(W_i) = \frac{\lambda b^{(2)}}{2(1-\hat{\rho})} - \sum_{i=1}^{N} \frac{\rho_i h_i^{(2)}}{2h_i} + \frac{\hat{\rho}\sigma_D^2}{2\overline{D}} + \frac{\overline{D}}{2(1-\hat{\rho})} \left[\hat{\rho} - \sum_{i=1}^{N} \rho_i^2 \right]$$
$$+ \sum_{i=1}^{N} h_i \boldsymbol{E}(L_i), \tag{13}$$

$$\boldsymbol{E}(L_i) = \Phi_i'(1) - \frac{\lambda_i(1-\rho_i)\overline{D}}{1-\hat{\rho}}, \tag{14}$$

where we have used (3)-(4) and $\boldsymbol{E}(K_i) = \Phi_i'(1)$ for derivation of (14), see Boxma (1989) and Katayama et al. (2001). Hence, arranging (12), (13) and (14), we reach (7). □

We also give an explicit mean waiting time formula for the symmetric polling system:

$$\boldsymbol{E}(W) = \frac{1}{2\left[1 - \hat{\rho} - \lambda g \overline{D}\{1 - H^*(\alpha)\}\right]} \left[\frac{\hat{\rho} g h^{(2)} + g^{(2)} h^2}{gh} + (N-1)\frac{g^{[2]}h}{g} \right.$$
$$\left. + (1-\hat{\rho})\frac{\sigma_D^2}{\overline{D}} + \overline{D}\left\{ 1 + \lambda g h + 2\lambda g \frac{d}{d\alpha} H^*(\alpha) + \frac{g^{(2)}}{g}(1 - H^*(\alpha)) \right\} \right], \tag{15}$$

where $g := g_i,\ g^{(2)} := g_i^{(2)},\ g^{[2]} := g_{i,j},\quad i, j = 1, 2, \cdots, N\ (i \neq j)$.

Remark 4.1. The relationship (7) reduces to Eq.(9) in Theorem 1 in Katayama et al. [9] setting $g_i = 1, g_i^{(2)} = g_{i,j} = 0$ and $\lambda_i = \lambda g_i$. Similarly, the formula (15) to Eq. (18) in Theorem 2 in [9] setting $g = 1, g^{(2)} = g^{[2]} = 0$ and $\hat{\rho} = N\rho$. A necessary and sufficient condition for stability of the time-limited sercice polling system is given by:

$$\hat{\rho} < 1 \text{ and } \lambda_i \left[p_i h_i + \overline{p}_i(h_i + \overline{D}) \right] + \sum_{\substack{j=1 \\ (j \neq i)}}^{N} \rho_j < 1 \quad \text{for all } i, \tag{16}$$

where the term in the brackets in (16) represents the average time of a modified service time $H := p_i H_i + \overline{p}_i(H_i + \sum_{i=1}^N D_i)$. The similar relationship to (7) has not been obtained even for the $M/G/1$ polling system with *gated time-limited service*, see Remark 4.1 in [9].

5 A General Pseudo-Conservation Law

Combining Theorem 1 and previous results on the four basic service disciplines derived by Boxma (1989) and Chiarawongse and Srinivasan (1991) and on the Bernoulli service also by Boxma (1989), we provide the following general formula concerning with batch Poisson arrivals. Let Ex, Ga, Lt, De, Be and Te denote the index sets of exhaustive service, gated service, one-limited (non-exhaustive) service, one-decrementing (semi-exhaustive) service, Bernoulli service and exponential time-limited service queues, respectively.

Theorem2 (General pseudo-conservation law).
For a stable $M^X/G/1$ polling system with mixed service of Ex, Ga, Lt, De, Be and Te service disciplines, a pseudo-conservation law is given by:

$$
\begin{aligned}
&\sum_{i\in Ex,Ga} \rho_i \boldsymbol{E}(W_i) + \sum_{i\in Lt} \rho_i \left[1 - \frac{\lambda_i \overline{D}}{1-\hat{\rho}}\right] \boldsymbol{E}(W_i) + \sum_{i\in De} \rho_i \left[1 - \frac{\lambda_i \overline{D}(1-\rho_i)}{1-\hat{\rho}}\right] \boldsymbol{E}(W_i) \\
&+ \sum_{i\in Be} \rho_i \left[1 - \frac{\lambda_i \overline{D}}{1-\hat{\rho}} \overline{p}_i\right] \boldsymbol{E}(W_i) + \sum_{i\in Te} \rho_i \left[1 - \frac{\lambda_i \overline{D}}{1-\hat{\rho}} (1 - H_i^*(\alpha_i))\right] \boldsymbol{E}(W_i) \\
&= \frac{\lambda}{2(1-\hat{\rho})} \sum_{i=1}^{N} (\hat{\rho} g_i h_i^{(2)} + g_i^{(2)} h_i^2) + \frac{\lambda}{1-\hat{\rho}} \sum_{i=2}^{N} h_i \sum_{j=1}^{i-1} g_{i,j} h_j + \frac{\hat{\rho}\sigma_D^2}{2\overline{D}} \qquad (17) \\
&\quad + \frac{\overline{D}}{2(1-\hat{\rho})} \left[\hat{\rho} - \sum_{i=1}^{N} \rho_i^2\right] + \frac{\overline{D}}{2(1-\hat{\rho})} \Bigg[\sum_{i\in Ex,Ga} 2\rho_i^2 + \sum_{i\in Lt} \lambda g_i^{(2)} h_i \\
&\quad + \sum_{i\in De} \lambda[(1-2\rho_i) g_i^{(2)} h_i - (\lambda g_i)^2 g_i h_i h_i^{(2)}] + \sum_{i\in Be} \lambda \overline{p}_i h_i (2\rho_i g_i + g_i^{(2)}) \\
&\quad + \sum_{i\in Te} \lambda h_i [2\rho_i g_i + 2\lambda_i g_i \frac{d}{d\alpha_i} H_i^*(\alpha_i) + g_i^{(2)} (1 - H_i^*(\alpha_i))] \Bigg].
\end{aligned}
$$

Remark 5.1.
(1) In an $M^X/G/1$ cyclic-service system, the following necessary and sufficient conditions for system stability are known for the individual service disciplines: $\hat{\rho} < 1$ for the exhaustive and gated services, $\hat{\rho} < 1$ and $\lambda_i < (1-\hat{\rho})/\overline{D}$ for all i, for the one-limited service, which is equivalent to $\lambda_i \boldsymbol{E}(C) < 1/(1-\rho_i)$, where the term, $1/(1-\rho_i)$, represents the expected number of customers served during a 1-busy period at Q_i, see Takagi (1990) and Altman et. al. (1992).
(2) It can be seen from (17) that the index set Te is equivalent to the Be if and only if the ith Bernoulli parameter $p_i = e^{-\alpha_i h_i}$ and $H_i^*(s) = e^{-sh_i}$.

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時間制限式サービスのポーリングシステムの一般的な擬似保存則

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要約

１人の扱い者（サーバ）が巡回して、$N\,(\geqq 2)$ 種類の客（ジョブ、呼、パケット等）の各待ち行列（バッファ）のサービスを行う巡回形待ち行列システム（ポーリングシステム）における仕事量の一般的な擬似保存則を導いている。これは、情報通信システムにおけるシステム遅延の近似評価やサーバ（プロセッサ）のサービス規律の最適化の研究に応用されている。まず、客の到着が集団的な場合を含む複合ポアソン過程に従う$M^X/G/1$型ポーリングシステムにおいて、客種ごとに異なる最大サービス期間だけ各待ち行列でサービスを行う「時間制限式」のサービス規律の下に、擬似保存則を導いている（定理１）。次に、既に求められている５種類の代表的なサービス規律（全処理式、ゲート式、客数制限式、半全処理的、確率制御式）の結果と本論文で求めた時間制限式のサービス規律の結果を統合して、今迄に最も一般的な擬似保存則を導いている（定理２）。

キーワード：擬似保存則、ポーリングシステム、複合ポアソン過程、時間制限式サービス